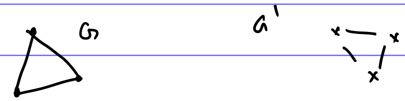
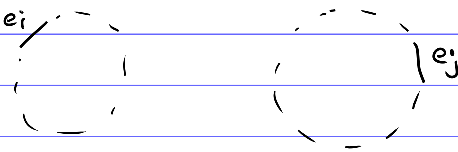
3. 

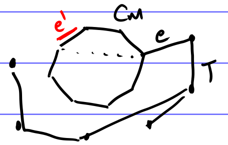
In C_n, any spanning tree has $T = n-1$ edges, so only one edge can be missing.

Let C_n have edges $\{e_1, \dots, e_n\}$. Will show e_i, e_j are adjacent.



Easy to see These have $n-2$ edges in common. Therefore G' is K_n.

b. $C_m \subset G$



Look at $G - V(C_m)$. If its connected, then take any tree T of $G - V(C_m)$, add an edge e from T to any one vertex of C_m. Then consider the family of trees

$$T \cup \{e\} \cup C_m \setminus e' \text{ where } e' \text{ is an edge of } C_m.$$

These are spanning trees, and by the previous part its clear K_m is a subgraph of G'

If $G - V(C_m)$ is not connected, then form spanning trees in each component of $G - V(C_m)$ and connect it up to a vertex in C_m. Again one obtains K_m as a subgraph.

c. P_n $G' = \{v\}$ a single vertex.

d. $|V(G')| \geq 2$. Suppose G has at least two spanning trees. T, T'
Recall the procedure that allows you to consider
$T - e + e'$ a new spanning tree (Prop 2.1.6)
Thus T and $T - e + e'$ are adjacent in G'

Suppose $|E(T) \cap E(T')| < n-2$

e. Take T and T'. Using Prob (2.1.6), remove e from T and add e' to T. $T - e + e'$.

Now $|E(T-e+e') \cap E(T')| > |E(T) \cap E(T')|$

So eventually we must get to T'.